Manifestations of Clustering in Binary and Ternary Fission of Low Excited Heavy Nuclei

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There comes a discussion of the evidences of preformation of a pair of magic nuclei (clusters) such as \(^{128}\)Sn, \(^{132}\)Sn, and \(^{70}\)Ni, \(^{82}\)Ge in the body of mother system at the early stage of its elongation. It is believed to be a crucial feature of the process of binary fission of low excited heavy nuclei. Strong experimental indication has been obtained that the fragment of conventional binary fission immediately after scission of the fissioning nucleus looks like a di–nuclear system consisting of the magic core and a light cluster to be some part of the neck. Similar clusters manifest themselves in the multibody decays observed for the first time.

1. Cluster nature of fission modes

1.1. Experimental and theoretical proofs

First attempts to create a model of the fission process involving shell aspects were taking place in early fifties of the last century. V.V. Vladimirski [1] was perhaps the first to have postulated that the fission probability had a noticeable value only if two cluster structures such as magic cores within the light and heavy fragment corresponding to the \(N=50\) and \(Z=50\), \(N=82\) shells were not destroyed. A dumbbell–like configuration consisting of two magic clusters connected by a flat cylindrical neck was considered as a typical shape of the fissioning
system. Formal representation of the fissioning system as a superposition of two interacting clusters (of arbitrary nucleon composition) is under exploration in a cluster model of fission developed by K. Wildermuth [2]. The outlined theory had unfortunately been “too basic” for practical use. There are also well known multiple calculations in the frame of two-center shell model approach [3, 4]. One of the most important and reproducible results obtained was to show that specific features of the level scheme of the fission fragments to be borne are already defined at relatively small distance between the centers of the clusters chosen for study. Thus, final fragments can grow from the cores along with elongation of the mother system. Visual proof of clustering of the fissioning system along the descent from the fission barrier was obtained in Refs. [5, 6]. Figure 1 shows potential energy of the fissioning nucleus $^{246}$Cm corresponding to the bottoms of the potential valleys as the function of parameter $Q$ proportional to the quadrupole moment of the system. The calculations were performed in the frame of Strutinsky approach in ten dimensional deformation space.

Fig. 1. Potential energy of the fissioning nucleus as a function of its elongation [6]. “Republished with permission of World Scientific Publishing Co., Inc., from “Fission potential energy surfaces in ten-dimensional deformation space”, V. Pashkevich, Y. Pyatkov, and A. Unzhakova, V. 18, No 4, 2009]; permission conveyed through Copyright Clearance Center, Inc.”
Both in valley of mass–asymmetrical shapes (3) and mass–symmetrical shapes (4) the system consists of the pairs of magic clusters (Sn/Ni, Sn/Ge and so on) and the left over nucleons forming a “neck” between the clusters. The map under discussion vividly demonstrates the role of the magic clusters in the body of the fissioning system in the form of fission modes. Actually, the ruptures occurred while the system descents along the distinct valley provide the fission fragments observed experimentally and treated as the appropriate fission mode [5]. Similar “visual” analysis of the nuclear shape in deformation process let authors of work [7] to reveal clustering in quasi–fission reactions leading to super–heavy systems.

Recent studies of the far asymmetric fission (Figure 2) also let us make inferences concerning the crucial role of just pair of magic clusters in fission process. As it was accentuated in Ref. [8]: “…fission is not only determined by the double shell closure in the heavy sphere of the scission point dumbbell configuration around \( A = 132 \) \((Z = 50, N \approx 82)\) but also by an effect of the double shell closure of \( Z = 28 \) and \( N \approx 50 \) in

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the corresponding light sphere”. As can be inferred from Figure 2, mass spectra of different actinide nuclei are bounded by mass ~70 amu. It was shown in Ref. [9] that those were really magic isotopes of Ni. The conclusion concerning the light mass boundary of mass spectra of actinides was confirmed as well by the results obtained in the course of studying of the (p, f) reaction [10].

1.2. Fine structures in mass–energy distributions of fission fragments

Specific manifestations of clustering were revealed as a result of processing of the FF mass–energy distributions using methods of image analysis [11].

Fig. 3. Fine structure of the FF mass–total kinetic energy distribution for the $^{233}\text{U}(n_{th}, f)$ reaction (a). Asymptotic borders of most pronounced “snake–like” structures observed indicate cluster origin of the whole image (b). Only light peak of the FF mass spectrum is analyzed here. We believe [12, 13] the FS under discussion to be the image of the distinct fission way extending “from the light cluster (Ge, Ga) to the heavy cluster (Sn)”. It is a new approach to extracting information from known mass–energy distributions of nuclear reaction products by direct processing of

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two–dimensional data [11]. For example, the typical $M–E$ (mass–energy) distribution of fission fragments in the $^{233}\text{U}(n_{th}, f)$ reaction seems to looks like a smooth hill (Figure 4a). The more detailed consideration reveals that any $E = \text{const}$ section of this distribution (Figure 4b) is not absolutely smooth but on the contrary, exhibits local irregularities (peaks) marked by arrows.

The origin of the peaks becomes clear from the following speculation. The yield $Y(M|E)$ of fission fragments with the mass $M$ and the fixed energy $E$ is

$$Y(M \mid E) = \sum_{Z} Y(M, Z \mid E).$$  \hspace{1cm} (1)

The summation in expression (1) is performed over all possible values of the nuclear charge $Z$ of the fission fragments. Thus, the spectrum shown in Figure 4b is a superposition of partial mass spectra for fixed charges (the so–called isotopic distributions) known from the experiment [14]. Let us define the term as a “fine structure”. By definition, this term means local regions (peaks) of the two–dimensional distribution with a yield higher than that on a smooth substrate.

![Fig. 4. The contour map of the experimental mass–energy FF distribution in the $^{233}\text{U}(n_{th}, f)$ reaction (a). $E–M$ cross section for the fragment energy $E = (100.5\pm0.5) \text{ MeV}$ [11]. Partial yields for fixed charges are shown by dotted lines.](image)

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As it follows from Figure 4b, the fine structure in this case is caused by a higher yield of fragments with an even charge (known odd–even modulation of mass yields due to the proton odd–even effect [15]). The peaks in the neighboring $E = \text{const}$ sections are correlated and they constitute regular structures in the $E–M$ plane in the form of ridges parallel to the axis of energy $E$ [14]. This structure will further be termed as vertical ridges.

![Fig. 5. The fine structure of the TKE–M distribution of $^{252}\text{Cf}$ fission fragments demonstrating the proton odd–even staggering. The FF nuclear charges are indicated near the ridges [5].](image)

An example of such fine structure in $^{252}\text{Cf}$ (sf) is shown in Figure 5 [5]. The position of the vertical ridges well agrees with the tops of the known isotopic distributions. At lower TKE values, i.e. at higher excitation energies in the scission point $E^* = Q–\text{TKE}$ (where $Q$ is the energy release for corresponding partition of $^{252}\text{Cf}$ and TKE is a total kinetic energy of the detected fragments), there appears an odd–even effect which is presumably associated with the complete clusterization of the fissioning nucleus at large elongations. In fact, the calculation of the potential energy surface (PES) of the fissioning $^{252}\text{Cf}$ nucleus predicts its specific shape at large elongations [5] (Figure 6a), that is the bulk of the nucleons is included into magic clusters of Sn, Mo, and Cd. The

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predictions well agree with the experimental data. The conditional mass–energy distribution $P(M|E^*)$ (Figure 6b) demonstrates a ridges leading to the partitions predicted in valley 3, 4.

Fig. 6. Potential energy of the fissioning nucleus $^{252}$Cf corresponding to the bottoms of the potential valleys, as the function of $Q$, proportional to its quadrupole moment. The panels depict the shapes of the fissioning system at points marked by arrows (a), “Republished with permission of World Scientific Publishing Co., Inc., from “Fission potential energy surfaces in ten-dimensional deformation space”, V. Pashkevich, Y. Pyatkov, and A. Unzhakova, V. 18, No 4, 2009; permission conveyed through Copyright Clearance Center, Inc.”; (b)—the contour map of the conditional mass–energy distribution $P(M|E^*)$. The panels depict the shapes of the fissioning system following from the PES calculations ascribed to the two dominant structures [5].

We have posed a question whether the structure in the mass–energy distribution of fission fragments that differs from vertical ridges produced by the odd–even modulation of mass yields, and is therefore caused by other physical reasons. For automatic suppression of vertical ridges in the fine structure search for, we have analyzed $M=\text{const}$ sections in which, as it is known [5, 17], the peaks can also be observed. In our studies, we used the algorithms for identifications of peaks known from gamma spectroscopy and from methods for processing noised images [11]. Figure 7 shows examples of a fine structure revealed in the

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total kinetic energy–mass distributions of fission fragments. The initial data were obtained using the time–of–flight spectrometers of fission fragments described in Refs. [18, 19]. Darker points in the tone diagram (Figure 7a) correspond to higher amplitude of the effect. There is only a light peak shown in the mass distribution of fission fragments. The symmetry of the structure shown in Figure 7b is caused by both the methods of measuring the mass of fission fragments (the method of two velocities [15]) and by the employed filter [20]. The prevailing fine structure is a sequence of snake like curves, sometimes having bifurcation points [11, 20].

What is the purpose of studying the debated fine structures? According to the existing concepts, in fission, for instance, the evolution of a decaying nuclear system is mainly determined by the potential energy surface (PES). Separate potential valleys in PES [21, 22] cause the existence of selected trajectories of the system in the deformation space. As it is shown in Ref. [23] in the frame of the time–dependent Hartree–Fock approach, the scission can take place at any point of descent of the system down the fission valley, which is detected as an event of fission in the space of experimentally observed variables.

In other words, a trajectory in the deformation space as a continuous sequence of nuclear states in the fission valley is mapped to continuous
trajectories (smooth curves) in the plane of experimentally observed variables [5], such as coordinates in which Figure 7a has been constructed, namely the total kinetic energy and the mass of fission fragments. Thus, we assume that the fine structure under discussion is a unique image of separate fission trajectories passed by a system for a time as $\sim 10^{-20}$ [24].

Special attention was paid to a quantitative estimate of the reliability of the revealed fine structures [25].

Fig. 8. Presumable shape of the fissioning nucleus along the descent from the fission barrier [12]*. See text for details.

Analyzing specific features of FS we have put forward a hypothesis concerning the shape of the decaying system along the descent from the fission barrier [24]. Presumably it looks like multibody nuclear molecule based on a pair of magic clusters (Ge/Sn in the most populated valley) while residual nucleons form torus--like “neck” in between gross clusters (Figure 8). The neck could be also clusterized i.e. consists of He isotopes and neutrons. The evident difference of the proposed compared to those seen in Figure 6 could be traced back to the fact that only simply connected shapes are adopted in the known calculations of PES [22, 23]. Thus, in the frame of the hypothesis under discussion the most compact configuration of the system looks like the one shown in Figure 8a. With further elongation the toroidal neck exports nucleons to the space between the tips of the gross clusters. Following [5] we may suppose that

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the scission of the system can come into being at any stage of the descent (actually according to [5] it could be tunneling of the system into the valley of separated fragments). As a result, the scission results in forming of two fragments. Heavy fragment unites Sn cluster and central part of the semi–torus (this part is marked in Figure 8b by the dot line), while the light one consists of Ge cluster and residual part of the torus (marked by two arrows). The scenario is supposed to be a symmetric reference to gross clusters i.e. heavy fragment can be composed of the Sn cluster and the residual part of the semi–torus, while the Ge cluster joins its central part.

In our earlier work [26] a toroidal neck was supposed to be clusterised forming a circle consisted of α–particles and other isotopes of He.

Fig. 9. (color online) Shapes of the fissioning $^{252}$Cf nucleus for the parameter of the hexadecapole deformation $\beta_{40} = 1.7$ and $1.902$ [27]. Mass ratio for the middle and one of the extreme fragments is equal to 2.13 and 1.87, respectively (a). b) Self–consistent ground–state density of $^{28}$Si calculated in the framework of nuclear energy density functionals [28] vividly demonstrates cluster structure; "Reprinted figure with permission from: J.-P. Ebran, E. Khan, T. Nikšić, and D. Vretenar, Physical Review C, V. 83, page 044307–4, 2013; Copyright (2016) by the American Physical Society."

The shape of the fissioning nucleus depicted in Figure 8 seems to be extremely exotic compared to “classic” dumbbell–like shapes discussed elsewhere [22]. Nevertheless, recent theoretical calculations confirm an actuality of such consideration. A mechanism of true ternary fission was proposed in Ref. [27]. This mechanism is driven by a hexadecapole deformation of the fissioning nucleus. Typical shapes of the fissioning

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1 "Physics of Atomic Nuclei, Ternary fission of nuclei into comparable fragments, V. 78, No. 5, 2015, page 550, F.F. Karpeshin, with permission of Springer".
$^{252}$Cf nucleus are shown in Figure 9a. Torus like “neck” unites two other constituents in a single nucleus. The similar shape obtained in the framework of nuclear energy density functionals for $^{28}$Si nucleus is shown in Figure 9b [28].

1.3. Clustering of the fissioning system in third minimum of the fission barrier

The pronounced third minima have been predicted in theoretical studies of thorium and uranium isotopes, especially in those carried out with the macroscopic–microscopic (MM) approach [21, 29, 30]. These predictions were used for the interpretation of resonances observed experimentally. On the other hand, self–consistent studies based on the nuclear density functional theory, as well as recent MM works, typically find a third minimum that is much shallower. To clarify the situation, the self–consistent calculations were carried out in Ref. [31]. The authors used the finite–temperature superfluid nuclear density functional theory. They consider two Skyrme energy density functionals: a traditional functional SkM* and a recent functional UNEDF1 optimized for fission studies. The shape of the fission barrier obtained for $^{232}$Th is shown in Figure 10.

Fig. 10. Potential–energy curve for $^{232}$Th with SkM* obtained with a basis of 1140 (solid line) and 1771 (dashed line) basis states [31]; "Reprinted figure with permission from: J. D. McDonnell, W. Nazarewicz, and J. A. Sheikh, Physical Review C, V. 87, page 054327–2, 2013; Copyright (2016) by the American Physical Society."
The impressive evidences of preformation of magic clusters in the body of the mother fissioning system were demonstrated. The density profiles appeared to correspond to the third minimum of $^{232}$Th isotope shown in Figure 11. The paper demonstrates “that the third minimum can be associated with a dimolecular configuration involving the spherical doubly magic $^{132}$Sn and a lighter Zr or Mo fragment in a deformed configuration”. The nuclear compositions of the lighter fragments are also close to the magic ones [32]. The effect is reproduced for the series of Th and U isotopes (Figures 11, 12).

Fig. 11. (Color online) (top) Cross section of total density of $^{232}$Th in $yz$ plane calculated with (a) SkM* and (b) UNEDF1 at third minimum ($Q_{20} = 165$ b), compared to cross sections of $^{132}$Sn and $^{100}$Zr densities. (bottom) Density profiles for $^{232}$Th and the $^{132}$Sn and $^{100}$Zr fragments along the $z$ axis obtained in (c) SkM* and (d) UNEDF1 [31]; "Reprinted figure with permission from: J. D. McDonnell, W. Nazarewicz, and J. A. Sheikh, Physical Review C, V. 87, page 054327–3, 2013; Copyright (2016) by the American Physical Society."
Fig. 12. (Color online) Contour plots for total densities of (a) $^{228}$Th, (b) $^{228}$U, and (c) $^{232}$U calculated with UNEDF1 (solid lines) and SkM* (dashed lines) compared with fragment densities: spherical $^{132}$Sn and a lighter deformed nucleus around $^{100}$Zr. The contour levels shown are at 50%, 90%, and 95% of the saturation density ($\rho_0 = 0.16 \text{ fm}^{-1}$) [31]; "Reprinted figure with permission from: J. D. McDonnell, W. Nazarewicz, and J. A. Sheikh, Physical Review C, V. 87, page 054327–5, 2013; Copyright (2016) by the American Physical Society.".

The calculations for the super–heavy nuclei demonstrate similar tendency. For instance, Figure 13 shows three–humped fission barrier of $^{296}$Lv [33].

Fig. 13. Three–humped barrier calculated along the fission path of $^{296}$Lv [33].
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The authors conclude: “These intermediate minima correspond to the shape isomer states. From the analysis of the driving potential we may definitely conclude that these isomeric states are nothing else but the two-cluster configurations with magic or semi–magic cores surrounded by certain amount of shared nucleons”.

Summing up, one can state an exclusive role of the magic clusters in the mechanism of binary fission. It is substantially stronger assertion than traditional “influence of shell effects” because clustering presupposes space localization of the corresponding nuclear object. The question now arises of whether conventional dumbbell–like shape of the system is really adequate to the fission process or a more complicated multibody nuclear molecule (Figure 8) should be included into our speculation. It is believed that a new kind of ternary decay discussed below give the arguments in favor of the latter hypothesis.

2. Collinear cluster tri–partition

This section is devoted to the observation of a new kind of ternary decay of low–excited heavy nuclei. We have called this decay mode as a “collinear cluster tri–partition” (CCT) in view of the observed features of the effect showing that the decay partners fly apart almost collinearly, and at least one of them definitely has magic nucleon composition. CCT is observed together with conventional binary and ternary fission. It could be one of the rare fission modes, but for the time being it is not a matter–of–fact yet. There have actually been many years between the experimental discovery of the heavy ion radioactivity and the development of the recognized theory of the process.

At the early stage of our work a process of the “true ternary fission”, the fission of the nucleus into three fragments of comparable masses was considered to be undiscovered regarding low excited heavy nuclei. Another possible prototype—three body cluster radioactivity—was also unknown. The so called “polar emission” [34] stands the closest, at least cinematically, to the CCT phenomenon, but only very light ions (up to isotopes of Be) have been observed so far.
From the theoretical point of view there are numerous indications on possible ternary decays of the low excited heavy nuclei with comparable masses of the decay products. Within the framework of the liquid drop model (LDM) Swiatecki [35] has shown that the fission into three heavy fragments in energetically more favorable than binary fission for all nuclei with fission parameters $30.5 < Z^2/A < 43.3$. In 1963 Strutinsky [36] has calculated the equilibrium shapes of the fissioning nucleus and showed that along with the ordinary configuration with one neck, there was the possibility of more complicated elongated configurations with two and even three necks, at the same time it was emphasized, that such configurations were much less probable. Later Diehl and Greiner [37, 38] showed a preference for prolate over oblate saddle-point shapes for the fission of a nucleus into three fragments of similar size. Such pre–scission configurations could lead to almost collinear separation of the decay partners, at least in a sequential fission process. The results demonstrating a crucial role of shell effects in the formation of the multi–body chain–like nuclear molecules were obtained by Poenaru et al. [39]. We also would like to refer to the recent theoretical articles devoted to unusual ternary decays of heavy nuclei including CCT [40–46]. The authors analyze the potential energy of different pre–scission configurations leading to ternary decays, and the kinetic energies of the CCT partners [47] are calculated for a sequential decay process. These results can be considered as the first step in the description of the CCT process.

Having in mind both theoretical and experimental background mentioned above we have come to the conclusion, that the collinear tri–partition of low–excited heavy nuclear systems would be a promising field of research. The basic results obtained so far are represented here.

2.1. The most pronounced manifestation of the CCT

The bulk of our results has been obtained within the framework of the “missing–mass” approach. With the use two–arm spectrometers there have been measured binary coincidences. This means that only two fragments were actually detected in each fission event, and their total mass, the sum $M_s$ will serve as a sign of multi–body decay in case it
appears to be significantly smaller than the mass of the initial system. The registration of a third fragment was blocked, as explained below and shown in Figure 14.

Fig. 14. Scheme for coincidence measurements of two fission fragments of $^{252}$Cf (sf). The experiment has been performed at the FOBOS setup [48]. Here: 1—Cf source, 2—source backing, 3—micro–channel plate (MCP) based timing “start” detector, 4—position sensitive avalanche counter (PSAC) as “stop” detector, 5—ionization chamber (BIC) with the supporting mesh, 6—the mesh of the entrance window. The front view of the mesh is shown in the insert a), an enlarged mesh section is presented in the insert b). After passage of the two fragments through the source backing, two light fragments $L_1$ and $L_2$, are obtained with a small angle divergence due to multiple scattering. In b) we show that one of the fragments ($L_1$) can be lost hitting the metal structure of the mesh, while the fragment $L_2$ reaches the detectors of the arm1. The source backing (2) exists only on one side and causes the mentioned angular dispersion in the direction towards the right arm1 [49].

Fig. 15. a) Contour map (in logarithmic scale, the steps between the lines are approximately factor 2.5) of the mass–mass distribution of the collinear fragments of $^{252}$Cf (sf), detected in coincidence in the two opposite arms of the FOBOS spectrometer. The specific bump in arm1 is marked by an arrow. b) Projection of the “Ni”–bump onto $M_1$–axis obtained in three different experiments performed at the FOBOS spectrometer modules [49, 50].
Figure 15 shows in a logarithmic scale the two-dimensional distribution \([M_2-M_1]\) of the two registered masses of the coincident fragments [49]. In the experiment performed at the FOBOS setup \(M_1\) is defined as a fragment mass derived from the arm pointing towards the detector arm with the additional dispersive (scattering) materials. There were selected only collinear fission events with a relative angle of \(180\pm2^\circ\), which corresponds to the typical angular spread for conventional binary fission fragments. The “tails” in the mass distributions marked 3–6 in Figure 15a extending from the regions (1) and (2) which are used to mark the conventional binary fission, are mainly caused by the scattering of fragments on both the foils and on the grid edges of the “stop” avalanche counters and in the ionization chambers. We emphasize the small but important asymmetry in the experimental arrangement for the two arms, which consists of the thin source backing (50 μg/cm\(^2\) of Al\(_2\)O\(_3\)) of the target and the “start” detector foil located only in arm1 (Figure 15a). It is an astonishing difference in the counting rate and in the shapes of the “tails” (3) and (4) that draws our attention. In the case shown in Figure 15a there is a distinct bump, marked (7), on top of the latter “tails” (4). The bump is located in a region corresponding to a large “missing” mass. The statistical significance of the events in the structure (7) can be deduced from Figure 15b, where the spectra of the total (summed) masses \(M_s\) for the “tails” (4) and (3) are being compared. The yield of the events in the difference spectrum \(c\), is \((4.7\pm0.2)\times10^{-3}\) relative to the total number of events in the distribution shown in Figure 15a. It is only a lower limit of the yield because the following reason: if both fragments (\(L_1\) and \(L_2\) in Figure 14) pass on and enter into the BIC, we register a signal corresponding to the sum of the energies of the two fragments. In this case the event will be treated as binary fission with the unusual parameters. The existence of the bump with the similar yield was confirmed in two other experiments carried out at the modified FOBOS spectrometer [50].

As it was emphasized above a specific supporting grid in the FOBOS detecting modules provides the selection of the ternary events. At the same time it serves as a source of the scattered fragments forming the “tails” (Figure 15a) simulating the missing mass events. This effect is essentially suppressed in the mosaic COMETA spectrometer (Figure
16a) which basically allows detecting all the partners of the ternary decay.

Fig. 16. Scheme of the COMETA setup, which consists of two mosaics of eight PIN diodes each (4), MCP–based start detector (2) with the $^{252}$Cf source inside (1), and a “neutron belt” (3) consisting of 28 $^3$He–filled neutron counters in a moderator. The cross–section of the belt is marked by an arrow (a). b)—layout of the experiment for the neutron coincidences with the modified spectrometer based on FOBOS detector modules (1), a “neutron belt” consisting of 140 $^3$He–filled neutron counters (2), and a “start” avalanche counter with the Cf source inside (3).

This methodically quite different experiment confirms our previous observations concerning the structures in the missing mass distributions. In this case there is no tail caused by the scattering from material in front of the $E$–detectors. Figure 17a shows the region of the mass distribution for the FFs from $^{252}$Cf (sf) around the “Ni”–bump ($M_1 = 68–80$ amu, $M_2 = 128–150$ amu). The structures are seen in the spectrometer arm facing the source backing only. No additional selection of the fission events has been applied in this case which resulted in the experiment having almost no background. A rectangular–like structure below the locus of binary fission is bounded by magic nuclei (their masses are marked by the numbered arrows) namely $^{128}$Sn (1), $^{68}$Ni (2) and $^{72}$Ni (3). In Figure 17b we show the projection of the linear structure seen at the masses of 68 and 72 amu. Two tilted diagonal lines with $M_s = 196$ amu and $M_s = 202$ amu (marked by number 4) start from the partitions 68/128 and 68/134 (all the nuclei are magic), respectively.
Fig. 17. Region of the mass–mass distribution for the FFs from $^{252}$Cf (sf) around the CCT–bump (similar to that marked by an arrow in Figure 15a). The result was obtained at the COMETA setup. No additional gates were applied. The background of the scattered fragments is very low due to the use of PIN–diodes (absence of a grid before the surface). An internal structure of the “bump” seen as horizontal lines from points (marked by arrows, 2 and 3) is shown in Figure 17b as a projection.

The result for the measured charges (Figure 18) confirms the previous findings with the mass distributions namely the existence of an additional bump linked with Ni isotopes (“Ni”–bump) in the arm with the scattering medium.

One of the decay modes which contribute to the bump and manifests itself by the tilted ridges $M_1 + M_2 = \text{const}$ can be treated as a new type of cluster decay compared to a well–known heavy ion or lead radioactivity. The key features of both of them are summed up in Figure 19. The relatively high CCT yield can be well understood once you visualize the collective motion through the hyper–deformed pre–scission shapes of the mother systems, which is supported by the fact that the linear arrangement uses the lowest Coulomb potential energies of three clusters. We also emphasize, that the $Q$ values for ternary fission are more positive by 25–30 MeV, which again is due to the formation of magic fragments like in binary fission. The ternary fission process must be considered to proceed sequentially, with two neck ruptures in a short time sequence.
Fig. 18. Nuclear charge spectra for the FF from the reaction $^{235}\text{U}(n_{th}, f)$, the FF are detected in the two opposite spectrometer arms. A difference in the yields (bump) presented in the upper panel in a linear scale is visible for the charges around $Z = 28$ (isotopes of Ni) [49].

Fig. 19. Cluster scheme for the comparison of the lead radioactivity with collinear cluster tri–partition [49].


2.2. Conclusions from the neutron gated data

The results of our previous experiments allow us to assume that there are several CCT modes [51] with the middle fragment of the three–body pre–scission chain with very low velocity after the scission takes place. Such fragments are expected to emit neutrons almost isotropically. The neutrons emitted from the moving binary fission fragments are focused predominantly along the fission axis. In order to exploit this difference for revealing the CCT events, there was assembled a “neutron belt” in a plane perpendicular to the symmetry axis of the spectrometer (Figure 16). According to modeling and the previous experiments, the detection efficiency is estimated to be ~5% and ~12% for a neutron emitted in binary fission and from an isotropic source, respectively. The array of neutron counters of the similar geometry was exploited earlier at the modified FOBOS spectrometer [50]. One of the results obtained at the COMETA setup is shown in Figure 20.

Fig. 20. Rectangular structures bounded by the magic clusters in the FFs mass correlation distributions obtained at the modified FOBOS spectrometer (a) and COMETA setup (b). See Figure 16. Selection by the experimental neutron multiplicity was used [50].

As can be inferred from the figure, the rectangular structure visible in its upper right hand corner is bounded by the nuclei with the masses in the vicinity of known magic nuclei (shown in brackets). These masses (except for double magic $^{132}\text{Sn}$) were calculated based on the unchanged charge density hypothesis for the fission of $^{252}\text{Cf}$ nucleus. We actually known that at least three neutrons were emitted in each fission event.
presented in the figure. A change in the nuclear composition of the mother system can lead to a shift of the masses of the magic nuclei if neutrons were emitted from the decaying system (pre–scission neutrons). This is likely what we observe here. As to the upper right hand corner of the rectangle both mass and charge conservation laws are found only if the upper side of the rectangle corresponds to $^{109}_{43}$Tc nucleus while the mass of 140 amu corresponds to the nucleon combination of $^{140}_{55}$Cs.

The structure manifests itself exclusively owing to the difference of the neutron sources for the fragments appearing in both binary fission and CCT, respectively. These two decay modes must differ in the neutron multiplicity or/and in their angular distributions of the emitted neutrons in order to provide higher registration efficiency for neutrons linked with the CCT channel. At the same time the excitation energy of the system at the scission point defined as $E_{ex} = Q - TKE$ (where $Q$ is the reaction energy, and TKE is the total kinetic energy of all the decay fragments) is known from our experimental data. It does not exceed $E_{ex} = 30$ MeV. This value of the excitation energy is high enough to allow the emission of three or four neutrons, which corresponds to almost the mean neutron multiplicity of binary fission. Thus, the neutron source linked with the new CCT channels must have a much smaller velocity compared to conventional binary fragments, or it can be almost at rest in the extreme case [47]. The latter agrees with the foregoing hypothesis stating that we deal with the pre–scission neutrons, at least, with those in very light missing masses.

Thus, the neutron gated data confirm an existence of the rectangular structures bounded by the magic nuclei in the FF correlation mass distributions. The analysis of the structures allows us to suppose that at least some of the CCT modes are linked with the emission of four pre–scission neutrons on the average.

2.3. Ternary decays with comparable masses of the fragments

The effective cleaning of the FF mass–mass distribution from the background linked with the scattered fragments was achieved by way of applying different selection gates. There was a specific rectangular structure revealed (Figure 21) when the events characterized by the
approximately equal momenta and velocities were selected in the mass–mass distribution of the FFs from $^{252}$Cf (sf). The structure is bounded by magic isotopes of $^{68}$Ni and $^{85}$As [52]. The results of different processing of the same are shown in Figure 22. Thus, the rectangular structure that is well seen in Figure 21 is mainly reproduced within different selection conditions providing additional information on the features of the events involved.

**Fig. 21.** Correlation mass distribution of the FFs with approximately equal momenta and velocities measured at FOBOS setup (a). Rectangular structure marked by an arrow is presented in section (b) in a larger scale.

**Fig. 22.** Results of another gating of the same initial data as those used for obtaining of Figure 21. Selection by both the FF momenta out of the tails linked with the scattered FFs and experimental neutron multiplicity $n \geq 1$ (a). The same distribution under additional condition of approximately equal FFs velocities (b). The condition $M_1 + M_2 = \text{const}$ is met along the titled line. It means that the missing mass is also fixed and is equal to 88 amu (presumably magic $^{88}$Se cluster) in this case. Lines in grey are draw to guide the eye.
The events characterized by approximately equal momenta and velocities were selected as well in the mass–mass distribution of the FFs from $^{235}$U(n$_{th}$, f) reaction. The revealed structure looks like the right angle whose vertex lies on the diagonal of the plot in the vicinity of the points (68, 68) amu (Figure 23a). The assumption that the sides of the right angle in Figure 23a are connected with magic Ni isotopes is confirmed by Figure 23b. It depicts the distribution of FFs with approximately equal momenta, velocities, and nuclear charges.

![Figure 23](image)

Fig. 23. Mass distribution of coincident FFs from the $^{235}$U(n$_{th}$, f) reaction with approximately equal momenta and velocities a) and distribution of selected events b) with not only approximately equal momenta and velocities but also with similar nuclear charges; the events at the vertex of the right angle have charges of $Z \approx 28$; for the events lying on the slantwise straight line $M_e = \text{const}$ the "lost" mass corresponds to the deformed magic nucleus $^{118}$Pd.

Note, that although the distribution in Figure 23b includes just a few events, the reliability of their analysis is quite high. Indeed, there are no other points in the vicinity of the concentration of points under consideration. Each point results from the measurement of six independent experimental parameters (time–of–flight, energy and drift time at each spectrometer arm).

A less stringent selection made it possible to reveal the whole family of events with comparable masses of the fragments. Figure 24a shows the distribution of FFs upon momentum and drift–time selections [53].

The most intense concentrations of points in Figure 24a remained from the loci of conventional binary fission. The number of points indicated by the arrow and situated on the straight line $M_1 = M_2$ increases
in line with statistics, and the total yield of these events was $8 \times 10^{-6}$ per binary fission.

Fig. 24. a) Mass–mass distribution of FF selected by the momentum and drift time. The arrow points to the group of events with approximately equal masses. b) Those events in the larger scale. The numbers label presumably the following pre–scission configurations based on magic clusters: (1) $^{78}\text{Zn}_{30} - ^{80}\text{Ge}_{50} - ^{77}\text{Zn}$, (2) $^{82}\text{Ge}_{50} - ^{72}\text{Ni}_{28} - ^{82}\text{Ge}$, (3) $^{88}\text{Se}_{34} - ^{60}\text{Ca}_{12} - ^{80}\text{Se}$, (4) $^{87}\text{Rb}_{38} - ^{46}\text{Ar}_{28} - ^{85}\text{Rb}$, (5) $^{102}\text{Zr}_{40} - ^{30}\text{Mg}_{24} - ^{100}\text{Zr}$, and (6) $^{108}\text{Mo}_{56} - ^{20}\text{O} - ^{108}\text{Mo}$ [53]; "Republished with permission of Pleiades Publishing, Ltd, from "Collinear cluster tripartition channel in the reaction $^{235}\text{U}(n, f)$", Yu.V. Pyatkov, D.V. Kamanin, Yu.N. Kopach, A.A. Alexandrov, I.A. Alexandrova, S.B. Borzakov, Yu.N. Voronov, V.E. Zhuchko, E.A. Kuznetsova, Ts. Panteleev, and A. N. Tyukavkin, V. 73, No 8, 2010".

2.4. Results from the triple coincidences

Evidently, for almost collinear decay partners a probability of their independent detection even by the spectrometer of high granularity decreases rapidly along with multiplicity. Nevertheless, ternary events caused by multi–body decays were detected in our experiments at the mosaic spectrometers. For instance, ternary events observed in the reaction $^{238}\text{U} + ^4\text{He}$ (40 MeV) [54] are given below. In the following three events magic or double magic Sn nuclei were detected as the heaviest fragments (Table 1). The corresponding light fragment (deformed magic nucleus) was clusterized in the scission point forming a di–nuclear system. Presumably, its brake–up comes into beings due to inelastic scattering on the material of start–detector. Such hypothesis is based on the fact that a momentum conservation law is not met in either of the three events.
In the next set of events the decaying system was also fully clusterised i.e. its mass was exhausted by two magic constituents. The “initial” clusters undergo fragmentation leading to formation of two di–nuclear systems. It contrasts with the previous case where both “initial” clusters are relatively soft deformed nuclei. Some examples of the events under discussion are given in Table 2. The de facto undetected fragments are highlighted in bold.

Table 1. Mass conservation law is met in the events presented

<table>
<thead>
<tr>
<th>Point number</th>
<th>Decay scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$^{128}\text{Sn} + ^{32}\text{Mg} + ^{80}\text{Ge} + 2\text{n}$</td>
</tr>
<tr>
<td></td>
<td>$^{112}\text{Ru}$</td>
</tr>
<tr>
<td>2</td>
<td>$^{132}\text{Sn} + ^{68}\text{Ni} + ^{42}\text{S}$</td>
</tr>
<tr>
<td></td>
<td>$^{110}\text{Ru}$</td>
</tr>
<tr>
<td>3</td>
<td>$^{130}\text{Sn} + ^{72}\text{Ni} + ^{40}\text{S}$</td>
</tr>
<tr>
<td></td>
<td>$^{112}\text{Ru}$</td>
</tr>
</tbody>
</table>

Table 2. Di–nuclear systems based on deformed magic constituents

<table>
<thead>
<tr>
<th>Point number</th>
<th>Decay scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$^{121}\text{Ag} + ^{23}\text{F} + ^{65}\text{Mn} + ^{31}\text{Al}$</td>
</tr>
<tr>
<td></td>
<td>$^{144}\text{Ba}$</td>
</tr>
<tr>
<td></td>
<td>$^{95}\text{Sr}$</td>
</tr>
<tr>
<td>2</td>
<td>$^{113}\text{Ru} + ^{31}\text{Mg} + ^{78}\text{Ni} + ^{20}\text{Ne}$</td>
</tr>
<tr>
<td></td>
<td>$^{144}\text{Ba}$</td>
</tr>
<tr>
<td></td>
<td>$^{96}\text{Sr}$</td>
</tr>
<tr>
<td>3</td>
<td>$^{130}\text{Sn} + ^{14}\text{C} + ^{62}\text{Cr} + ^{33}\text{Mg} + 3\text{n}$</td>
</tr>
<tr>
<td></td>
<td>$^{144}\text{Ba}$</td>
</tr>
<tr>
<td></td>
<td>$^{95}\text{Rb}$</td>
</tr>
<tr>
<td>4</td>
<td>$^{140}\text{Xe} + ^{25}\text{Ne} + ^{62}\text{Cr} + ^{13}\text{C}$</td>
</tr>
<tr>
<td></td>
<td>$^{165}\text{Gd}$</td>
</tr>
<tr>
<td>5</td>
<td>$^{134}\text{Te} + ^{30}\text{Ne} + ^{50}\text{Ca} + ^{27}\text{Ne} + \text{n}$</td>
</tr>
<tr>
<td></td>
<td>$^{164}\text{Gd}$</td>
</tr>
<tr>
<td>6</td>
<td>$^{110}\text{Te} + ^{11}\text{Be} + ^{62}\text{Cr} + ^{38}\text{V}$</td>
</tr>
<tr>
<td></td>
<td>$^{121}\text{Ag}$</td>
</tr>
</tbody>
</table>
3. Observation of shape isomer states in fission fragments

In previous section 2 we discussed the role of scattering medium in registration of the CCT products, where we stated that even if two CCT partners initially flew in the same direction in perfectly collinear line, they got some angular divergence after having passed the scattering medium on their flight pass due to multiple scattering. Thanks to such effect they can be registered independently in the “stop” mosaic detector, and even thin backing of the radioactive source provides the observable effect.

In order to increase the effect, the additional absorbers (metal foils) were introduced just behind the source at the distance of approximately 1 mm [55, 56] (Figure 25).

Fig. 25. Layout of the setup used in the experiment aimed at the study of the influence of the additional absorber on the characteristics of the CCT products. The shown double–armed spectrometer consists of the start detector with the Cf source inside and two mosaics eight PIN diodes each (a), b) \(^{252}\text{Cf}\) source and additional foils placed around.

The geometry of the source unit is shown in detail in Figure 25b. Additional absorber foil was placed at 1 mm distance from the Cf source. Typical fission fragment overcomes such distance in approximately 0.1 ns. There were ternary events analyzed. It means that three fragments were really detected in coincidence in three different silicon detectors. For the sake of convenience, the FFs from such events were labeled as \(M_1\), \(M_2\) and \(M_3\) in an order of decreasing masses in the ternary event. Mass distribution for ternary events is shown in Figure 26a. The total mass of the two heavier fragments \(M_1 + M_2\) is plotted vs. the mass of the lighter fragments \(M_3\). As can be inferred from the figure, in all the events
where the lightest fragment corresponds to the knocked out Ti ions \( (M_3 \text{ is around} 48 \text{ amu}) \) we observe a missing mass of fission fragments. It is a very astonishing fact. True, if the elastic Rutherford scattering of one of the fission fragments (FF) took place on the Ti foil, the total mass \( M_1 + M_2 \) would lie in the vicinity of the mass of the mother system namely 252 amu (taking into account emitted neutrons, approximately four on the average, and the experimental mass resolution). These values are marked in the figure by horizontal lines, while the events resulted from Rutherford scattering are expected to lie in the region marked by the dashed oval.

![Fig. 26](image)

**Fig. 26.** Mass distribution for ternary events. Ti foil 2.2 μm thick was used as the additional absorber (a). Mass spectrum of the fragments detected in coincidence with the ions knocked out from the foil and in the same arm with them. The masses of known magic nuclei are marked in the figure by arrows. (b) See text for details.

The mass spectrum of the fragments scattered in the foil and detected in coincidence with the knocked out ions is shown in Figure 26b. As can be inferred from the figure this spectrum differs from the FF mass distribution in conventional binary fission. The most impressive difference is observed at the yield of the magic \( ^{128}\text{Sn} \) nucleus.

The spectrum under discussion can be basically understood through the known manifestations of clustering in binary fission (see section 1). In accord with these views we suppose that the bulk of the fragments directly after scission of the mother system looks like a di-nuclear system consisting of the magic core and the light cluster which is to be some part of the neck. Actually, this system can be treated as a shape isomer state of the whole fragment.

Typical life–time of the shape isomers observed in our experiments exceeds 1 ns. The indication on the true life–time of these states can be
inferred from Ref. [57]. The search for isomeric $\gamma$ decays among fission fragments from 345 MeV/nucleon $^{238}$U has been performed at the RIKEN Nishina Center RI Beam Factory. In flight fission of a uranium beam has been used as the production reactions to populate isomers. After having been selected and identified by the superconducting in–flight separator BigRIPS the fission fragments were implanted in an aluminum stopper. The fast isotopic separation and identification of reaction products, which take place in several hundred nanoseconds, allow the event–by–event detection of isomeric $\gamma$ rays at the focal plane of the separator with small decay losses in flight. A total of 54 microsecond isomers with half–lives of $\approx$0.1–10 $\mu$s were identified. At least some of the isomers observed were attributed to shape isomers. It is reasonable to suppose these microsecond isomers’ half–lives to be the upper limit for half–lives of the FF shape isomers’ states observed in our experiments.

Strong theoretical support for existence of the shape isomer states in fission fragments is also provided by recent calculations carried out by P. Möllner and his coworkers [58]. Potential energy surfaces as functions of spheroidal, hexadecapole, and axial asymmetry shape coordinates for several thousands of nuclei from $A = 31$ to $A = 290$ were calculated. The deformations and energies of all minima deeper than 0.2 MeV were tabulated. It gives a good possibility to identify nuclei for which a necessary condition for shape isomers occurs, namely multiple minima in the potential–energy surface. It is known that the life–time of the shape isomer depends on the overlap between the nuclear wave functions of the shape isomer and the ground state, on the excitation energy of the shape isomer, and on the height of the saddle separating the shape isomer and the ground state [58]. Unfortunately, the estimation of the shape isomer life–times is beyond the scope of the outlined work.

Bearing in mind the results of this section two different mechanisms leading to observation of missing mass treated as a label of multi–body decay are likely realized. The first one discussed in sec. 2.1 is connected with blocking of one of the fragments from ternary decay at the entrance of the “stop” detector. All three fragments are supposed to be born actually in ternary decay of the mother system. A brake–up of the fragment from the conventional binary fission outlined in this section can
also be a reason of observable “ternary” decay. Both mechanisms can stay behind the structures seen exclusively in the spectrometer arm faced to the source backing, but only “true” ternary decay can give rise the structures symmetric relative to the spectrometer arms (sec. 2.3).

4. Summary

Here we mainly present the recent experimental results demonstrating different manifestations of clustering in binary and ternary decays of low excited heavy nuclei. In contrast with the light alpha–cluster systems we deal with very heavy clusters to be magic nuclei with masses of up to 160 amu. The following conclusions can be drawn from the findings outlined above.

1. There are reliable evidences for the di–cluster mechanism for forming the fission modes in binary fission of actinides. Preformation of a pair of magic nuclei (clusters) such as $^{128, 132}$Sn and $^{70}$Ni, $^{82}$Ge in the body of mother system at the early stage of its elongation gives rise to this effect. The feature is also reproduced for the super–heavy fissioning systems.

2. In the mass correlation distribution of fission fragments (FFs) from $^{252}$Cf (sf) and $^{235}$U(nth, f) reaction in the region of the big missing masses of about ~50 amu we observe a specific domain of increased yields, what we call as “Ni–bump” by virtue of their presence in the bump of two ridges centered at mass numbers $A = 68, 72$ corresponded to magic isotopes of $^{68, 72}$Ni. The experimental yield of the events constituting the bump is more than $4*10^{-3}$ per binary fission.

3. Along with the linear structures $A = \text{const}$ the tilted ridges $A_1 + A_2 = \text{const}$ (the missing mass is constant) were revealed in the Ni–bump. The ridges starts from the partitions corresponded to the pairs of known magic nuclei $^{68, 128}$Sn, $^{68, 134}$Te et cetera. We treat these tilted ridges conditioned by collinear cluster tri–partition as a manifestation of the decay similar to heavy ion or lead radioactivity. In the case the chain of two magic nuclei plays a role of double magic $^{208}$Pb.
(4) True ternary fission (the fission with comparable masses of the fragments) events from both $^{252}$Cf (sf) and $^{235}$U(nth, f) reaction were observed with the yield of $\sim 10^{-5}$ per binary fission. At least two of the detected partners in each ternary event show magic nucleon composition.

(5) The unique rectangular structures bounded by magic nuclei were revealed in the FF mass correlation plots gated by neutrons detected in the “neutron belt” sensitive to the isotropic component of the neutron emission.

(6) It was the first time the binary break–up of the fission fragment due to inelastic scattering in the metal foil had been observed. Based on the obtained results we may suppose the bulk of the fragments directly after scission of the mother system to look like a di–nuclear system consisting of the magic core, and the light cluster to be some part of the neck. This configuration can likely be treated as a shape isomer state of the whole fragment. The effect in hand brightly demonstrates an intimate link between conventional binary fission and ternary decays.

The summing up allows us to state that the evolution of the multicomponent nuclear molecule is strongly believed to be crucial for the low energy fission process both in binary and ternary channels.

Acknowledgements

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References

4. M.G. Mustafa, Transition from mass asymmetry to symmetry in the spontaneous
5. Yu.V. Pyatkov et al., Manifestation of clustering in the $^{252}$Cf (sf) and $^{248}$Cf(n$_{th}$, f)
6. V.V. Pashkevich, Yu.V. Pyatkov, A.V. Unzhakova, Fission potential energy surface
7. V. Zagrebaev and W. Greiner, Fusion and fission dynamics of heavy nuclear
(DANF2006)*, pp. 94–111, Smolenice Castle, Slovak Republic (Slovak Republic,
2006).
8. I. Tsekhanovich et al., Mass and charge distributions in the very asymmetric mass
9. D. Rochman et al., Super–asymmetric fission in the $^{245}$Cm(n$_{th}$, f) reaction at the
10. D.M. Gorodisskiy et al., Isotopic and isotonic effects in fission–fragment mass
11. Yu.V. Pyatkov et al., Manifestation of the Fine Structures in the fission fragment
mass–energy distribution in the $^{237}$U(n$_{th}$, f) reaction, *Nucl. Instrum. Methods A.* 488,
12. Yu.V. Pyatkov et al., Nontrivial manifestation of clustering in fission of heavy
nuclei at low and middle excitations, *Phys. of Atomic Nuclei.* 67(9), 1726–1730
(2004).
13. Yu.V. Pyatkov et al., Collinear cluster tri–partition of low excited heavy nuclei —
status and prospects of experimental investigations. In *Proc. of 17th Int. Seminar on
Interaction of Neutrons with Nuclei (ISINN–17),* pp. 150–157, Dubna, Russia
(Russia, 2009).
14. U. Quade et al., Nuclide yields of light fission products from thermal–neutron
induced fission of $^{235}$U at different kinetic energies, *Nucl. Phys. A.* 487(1), 1–36
(1988).
16. M.N. Rao et al., Determination of fission fragment angle by the methods of grid
pulse height and the time difference between grid and collector pulses in a back–to–
17. T. Ohtsuki et al., Binary structure in time distributions of fission fragments in 13–
18. A.A. Alexandrov et al., A time–of–flight spectrometer for unslowed fission
19. W.H. Trzaska et al., HENDES — high efficiency neutron detection system for
correlation measurements with HI beams. In *Proc. of the XIV Int. Conf. on the


32. H. Märtön, private communication.


44. W. von Oertzen, A.K. Nasirov, True ternary fission, the collinear decay into fragments of similar size in the \(^{252}\text{Cf}\) (sf) and \(^{235}\text{U}(\text{n}_{\text{th}}, \text{f})\) reactions, *Physics Letters B*. 734, 234–238 (2014).
51. Yu.V. Pyatkov et al., Collinear tripartition of \(^{248}\text{Cm}\) and \(^{252}\text{Cf}\) nuclei as a probe of clustering, in *Proc. of the Intern. Conf. of Nuclear Physics "Nuclear Shells—50 Years"*, pp. 144–150, Russia, Dubna (World Scientific, 2000).
53. Yu.V. Pyatkov et al., Collinear Cluster Tripartition Channel in the Reaction \(^{238}\text{U}(\text{n}_{\text{th}}, \text{f})\), *Physics of Atomic Nuclei*. 73(8), 1309–1316 (2010).